Querying Conceptual Schemata with Expressive Equality Constraints

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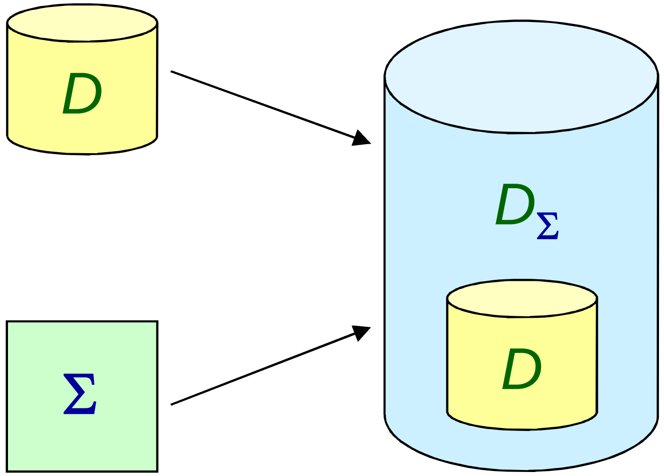
Ontological Database Management Systems



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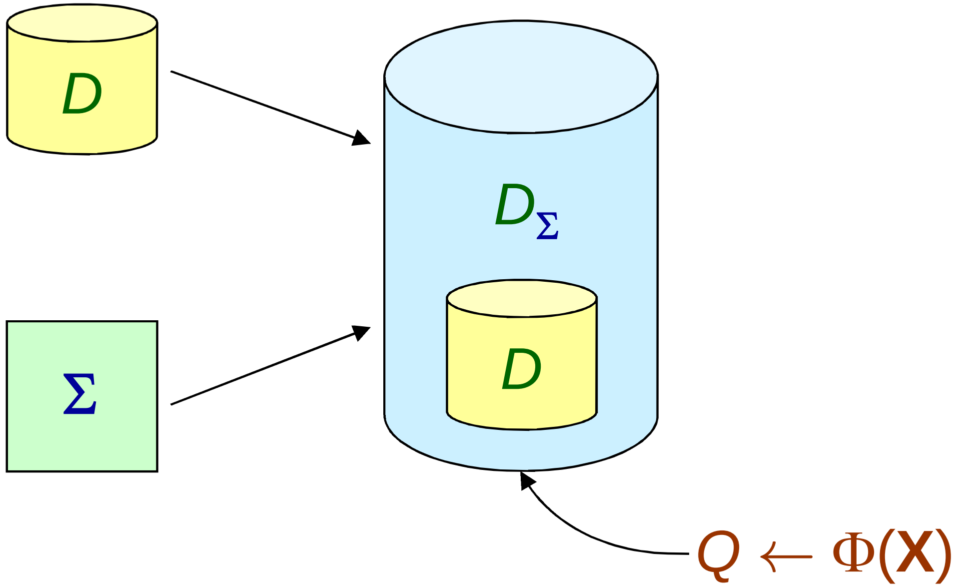
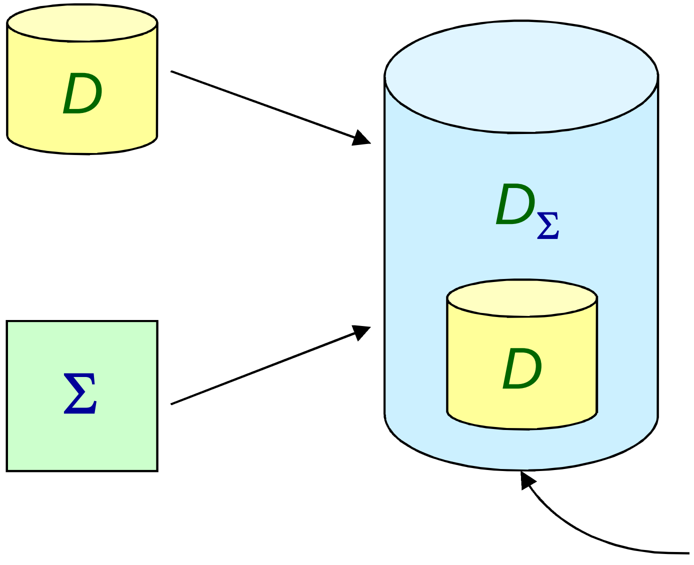


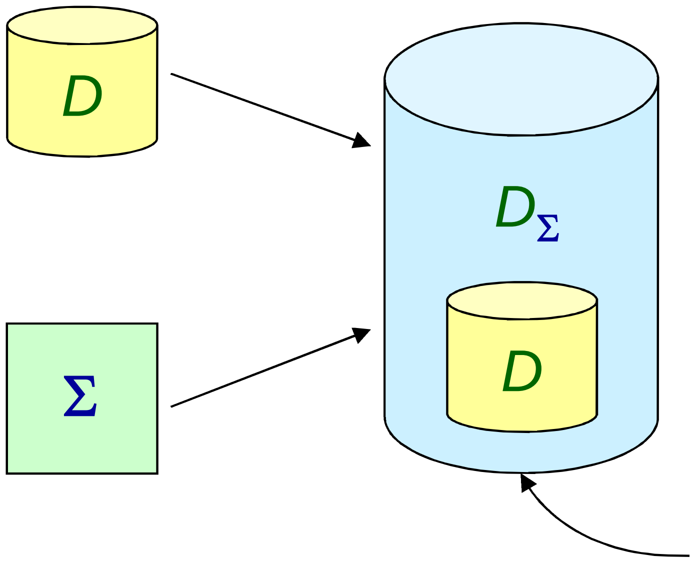
Ontological Database Management Systems



$$\Leftrightarrow \quad D \cup \Sigma \models Q \quad \Leftrightarrow \quad \forall B\,(\underbrace{B \models D \cup \Sigma}_{B \supseteq D \wedge B \models \Sigma} \;\Rightarrow\; B \models Q)$$

$Q \leftarrow \Phi(\mathbf{X})$ ✓

Ontological Query Answering



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$Q \leftarrow \Phi(\mathbf{X})$ ✓

$$D \cup \Sigma \models Q?$$

Ontological Query Answering: Example

D

person(john)

Σ

$\forall P\ person(P) \rightarrow \exists F\ father(F,P)$

$\forall F \forall P\ father(F,P) \rightarrow person(F)$

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$\forall B\ (B \vDash D \cup \Sigma)$

$\ldots\ father(z,\text{john})\ \ person(z)\ \ldots$

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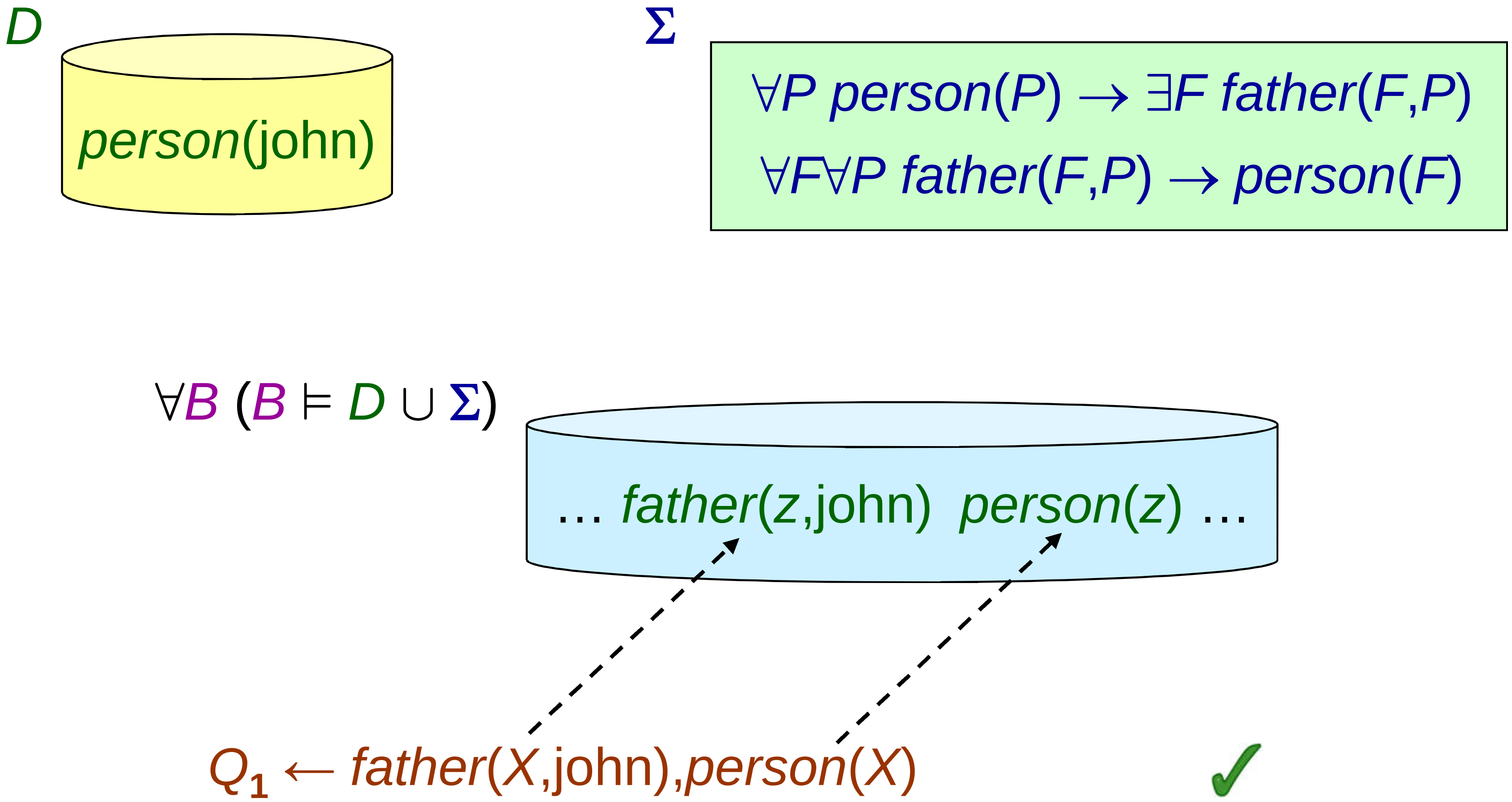

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$\forall B\ (B \vDash D \cup \Sigma)$

… *father*(*z*,john) *person*(*z*) …

$Q_1 \leftarrow father(X,\text{john}), person(X)$

✓

Ontological Query Answering: Example

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$\forall B\ (B \models D \cup \Sigma)$

$\ldots\ father(z,\text{john})\ \ person(z)\ \ldots$

$Q_1 \leftarrow father(X,\text{john}),person(X)$ ✓

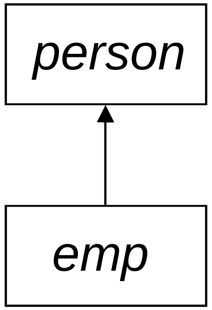
$Q_2 \leftarrow father(\text{john},X)$ ✗

Ontological Query Answering: Known Results

	Data Complexity	Combined Complexity
IDs	in AC_0 [Calì et al., IJCAI 03]	PSPACE-complete [Johnson & Klug, JCSS 84]
FKDs	in AC_0 [Calì et al., IJCAI 03]	PSPACE-complete [Johnson & Klug, JCSS 84]
DL-Lite	in AC_0 [Calvanese et al., JAR 07]	NP-complete [Calvanese et al., JAR 07]
$ER^{\pm}$	in AC_0 [Calì et al., ER 10]	PSPACE-complete [Calì et al., ER 10]

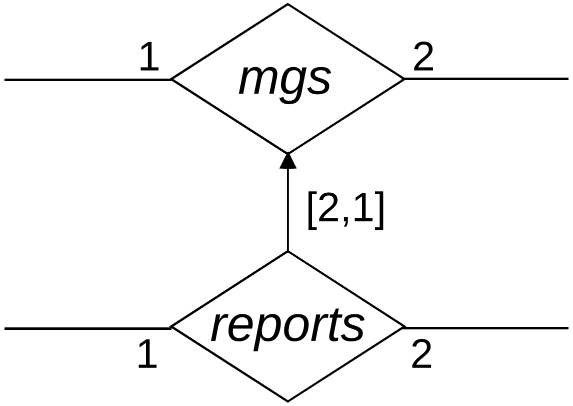
ER$^{\pm}$ Model: An Extended ER Formalism

IS-A among entities



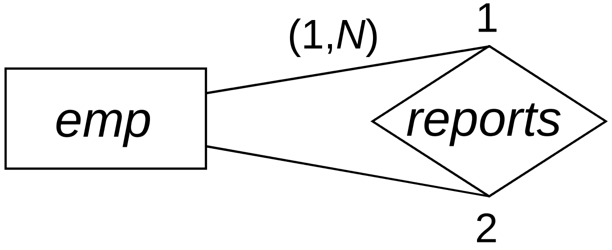
$$\forall X\ emp(X) \rightarrow person(X)$$

IS-A among relationships



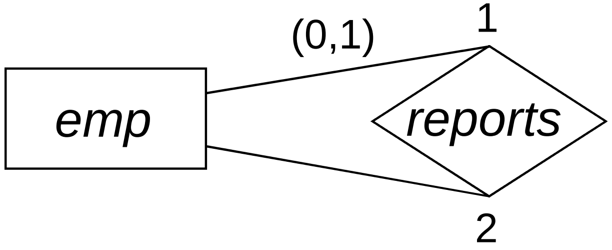
$$\forall X \forall Y\ reports(X,Y) \rightarrow mgs(Y,X)$$

Mandatory participation



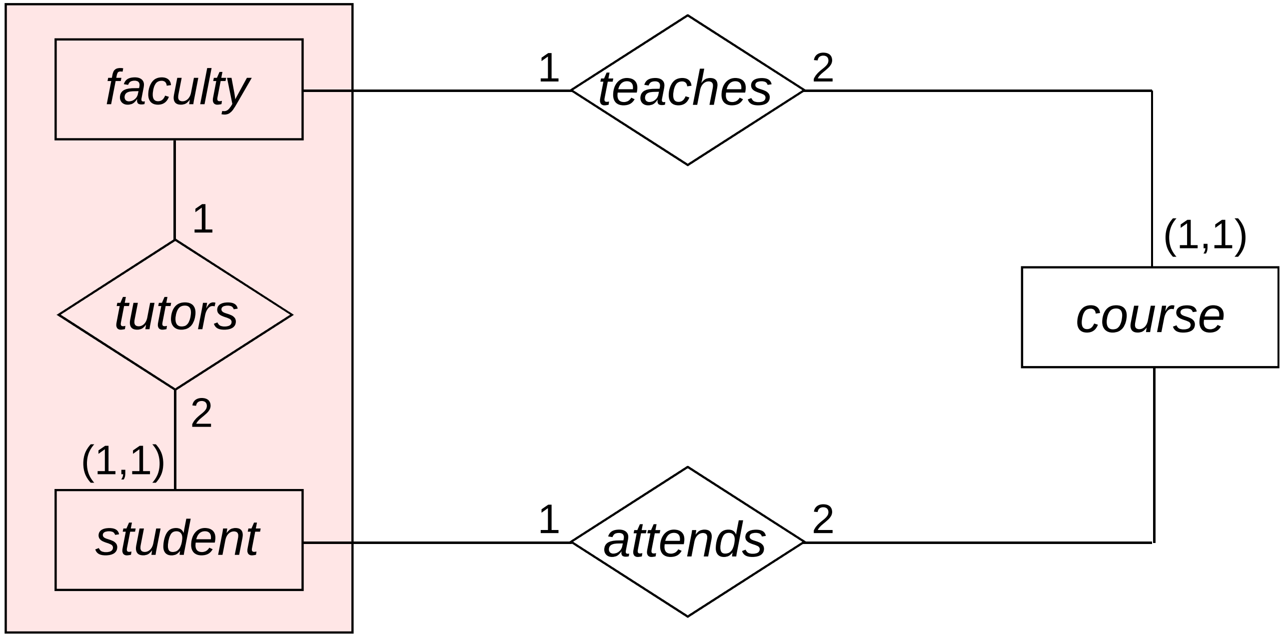
$$\forall X\ emp(X) \rightarrow \exists Y\ reports(X,Y)$$

Functional participation



$$\forall X \forall Y \forall Z\ reports(X,Y), reports(X,Z) \rightarrow Y = Z$$

ER$^{\pm}$ Model: A Motivating Example

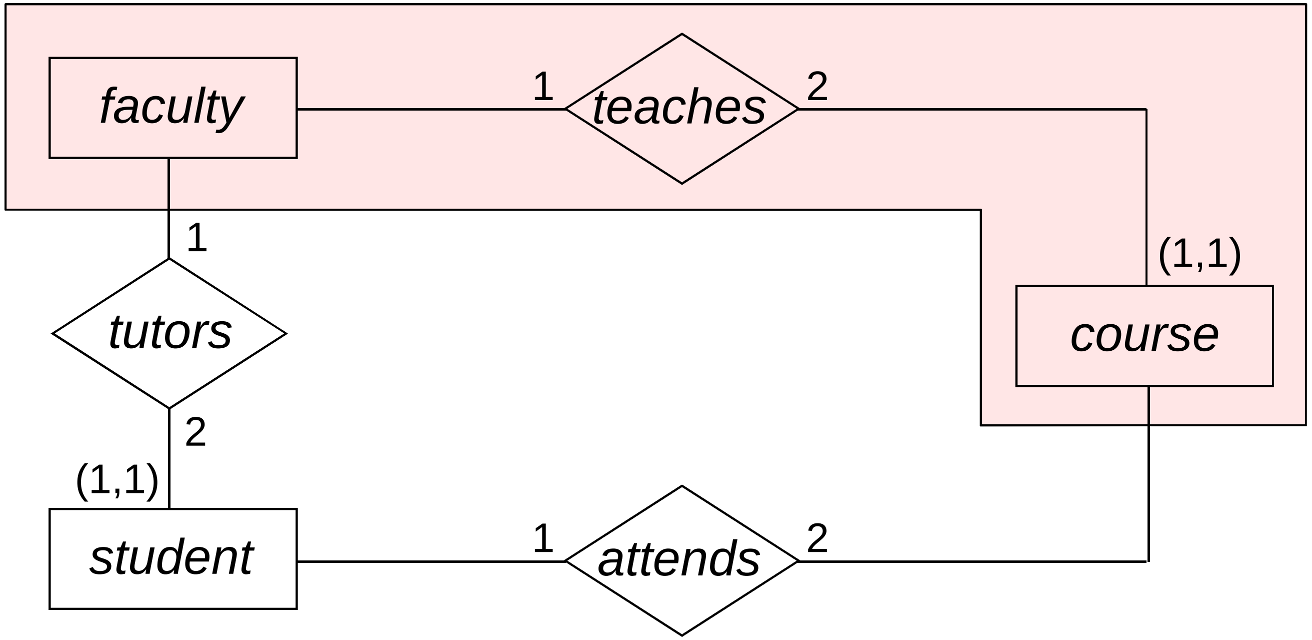


Each student is tutored by exactly one faculty member

$$\forall X\ student(X) \rightarrow \exists Y\ tutors(Y,X)$$

$$\forall X \forall Y \forall Z\ tutors(X,Y), tutors(X,Z) \rightarrow Y = Z$$

ER$^{\pm}$ Model: A Motivating Example

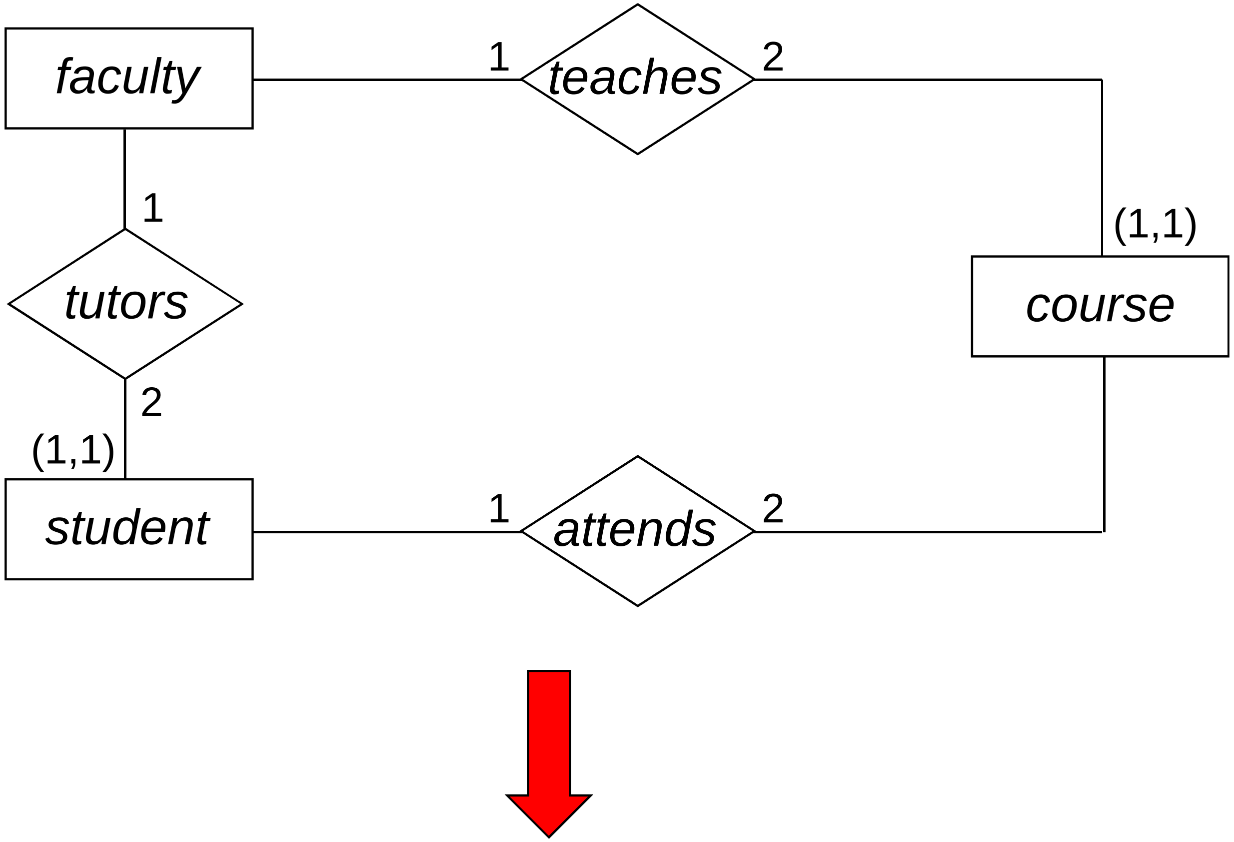


Each course is taught by exactly one faculty member

$$\forall X\ course(X) \rightarrow \exists Y\ teaches(Y,X)$$

$$\forall X \forall Y \forall Z\ teaches(X,Y), teaches(X,Z) \rightarrow Y = Z$$

ER$^{\pm}$ Model: A Motivating Example

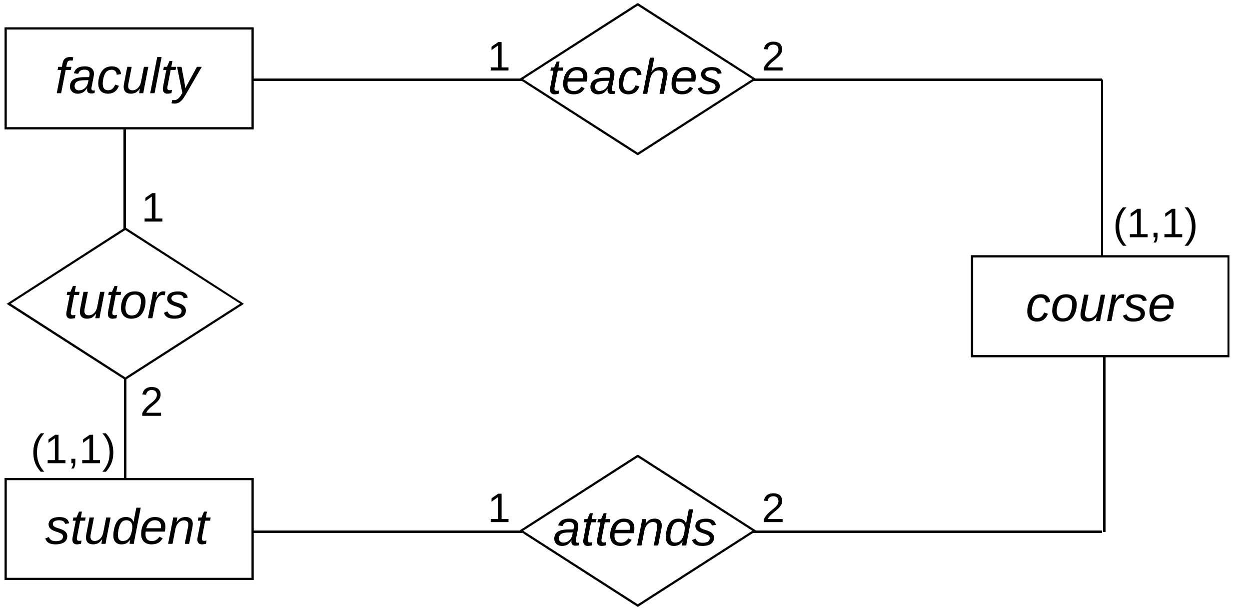


Linear Tuple-Generating Dependencies (TGDs) - $\forall \mathbf{X} \forall \mathbf{Y}\ p(\mathbf{X},\mathbf{Y}) \rightarrow \exists \mathbf{Z}\ s(\mathbf{X},\mathbf{Z})$

+

Key Dependencies (KDs) - $\forall \mathbf{X} \forall \mathbf{Y} \forall \mathbf{Z}\ p(\mathbf{X},\mathbf{Y}),p(\mathbf{X},\mathbf{Z}) \rightarrow Y = Z$

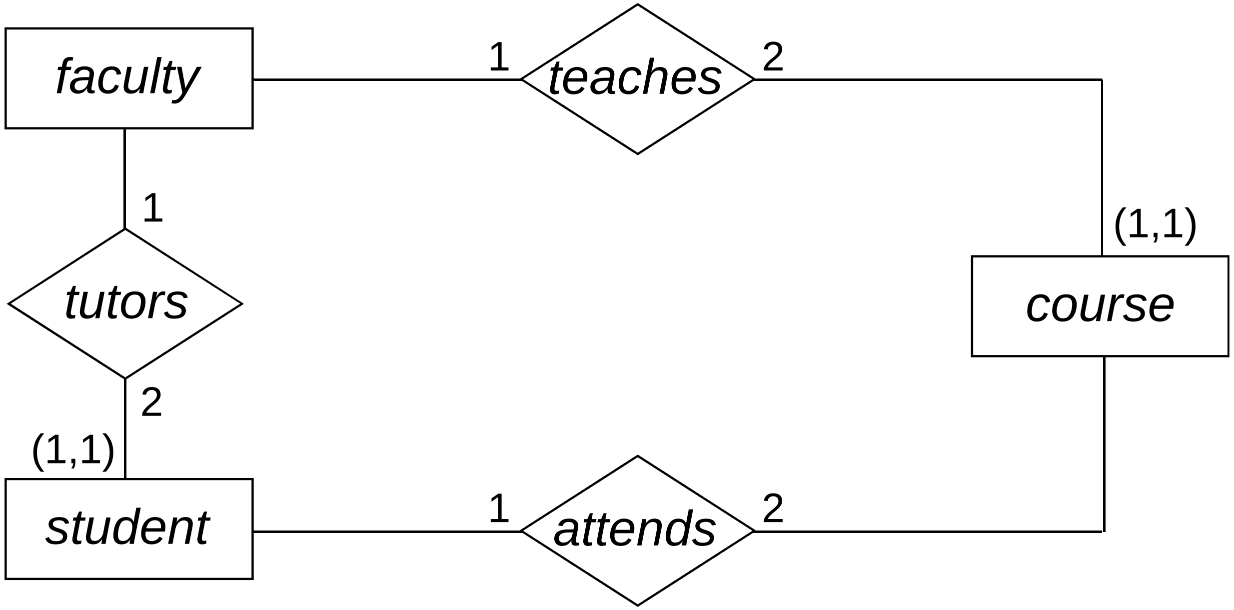
ER$^{\pm}$ Model: A Motivating Example



Each student can attend at most one course taught by his/her tutor

?

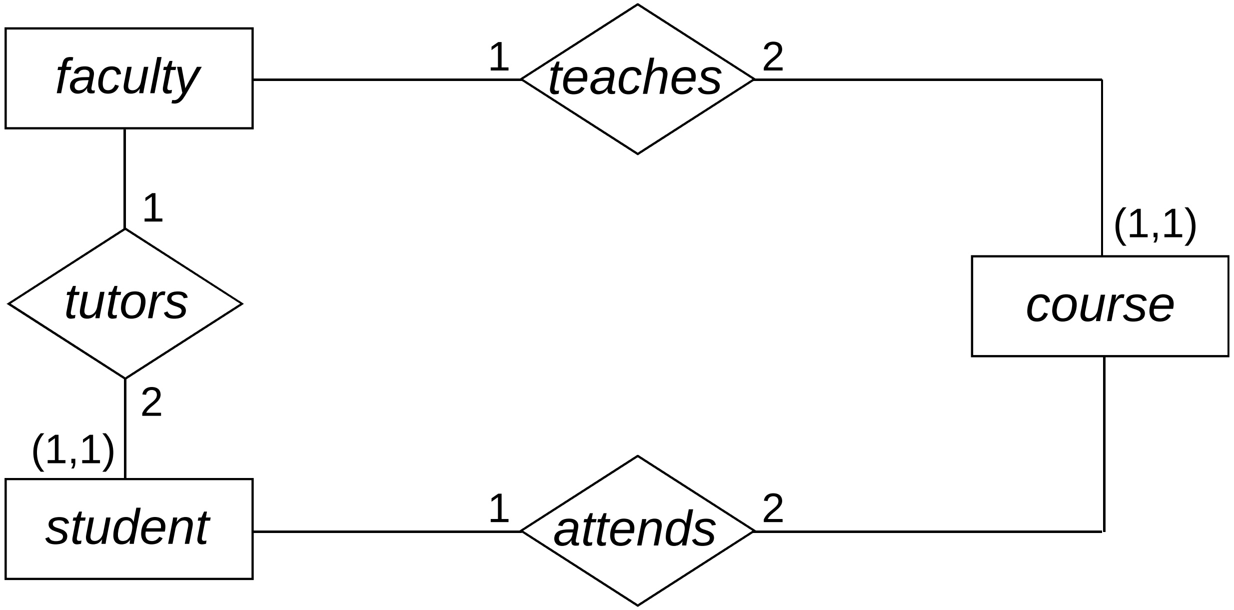
ER$^{\pm}$ Model: A Motivating Example



Each student can attend at most one course taught by his/her tutor

$$\forall S \forall C_1 \forall C_2 \forall F\ student(S), attends(S,C_1), attends(S,C_2),$$
$$tutors(F,S), teaches(F,C_1), teaches(F,C_2) \rightarrow C_1 = C_2$$

ER$^{\pm}$ Model: A Motivating Example



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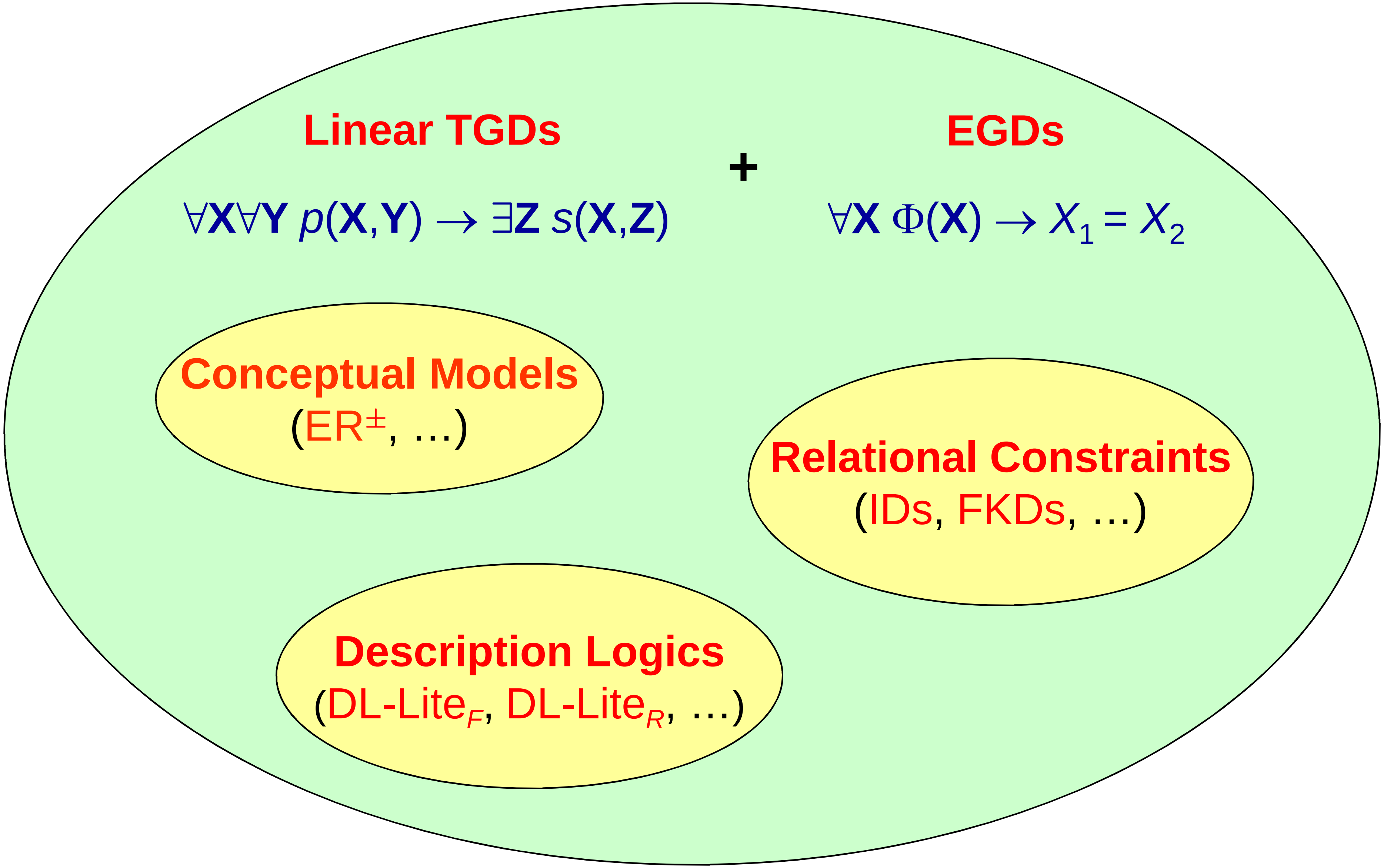
…we need equality-generating dependencies (EGDs)

Our Ultimate Goal



Linear TGDs

$\forall \mathbf{X} \forall \mathbf{Y}\; p(\mathbf{X},\mathbf{Y}) \rightarrow \exists \mathbf{Z}\; s(\mathbf{X},\mathbf{Z})$

+

EGDs

$\forall \mathbf{X}\; \Phi(\mathbf{X}) \rightarrow X_1 = X_2$

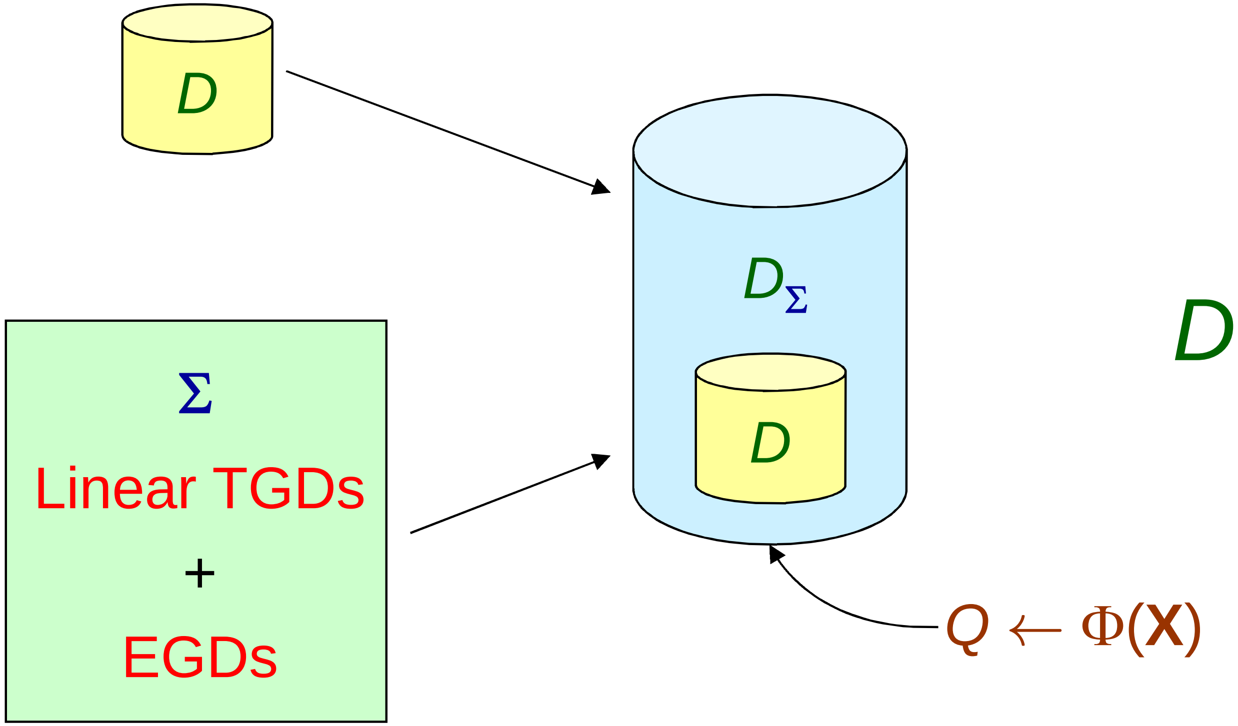
Conceptual Models
(ER$^{\pm}$, …)

Relational Constraints
(IDs, FKDs, …)

Description Logics
(DL-Lite$_F$, DL-Lite$_R$, …)

… without losing tractable data complexity

A Challenging Problem



$D \cup \Sigma \models Q$?

- Query answering under Linear TGDs and EGDs is undecidable

  [see, e.g., Chandra & Vardi, SIAM J. Comput. 1985]

- Linear TGDs and EGDs can be syntactically restricted?

The Chase Procedure

Input: Database D, set Σ of Linear TGDs and EGDs

Output: A model of $D \cup \Sigma$

D

$professor(\mathrm{p})$ $works(\mathrm{p},\mathrm{g})$

Σ

$professor(X) \rightarrow \exists Y\, leads(X,Y)$

$leads(X,Y) \rightarrow works(X,Y)$

$works(X,Y), works(X,Z) \rightarrow Y = Z$

$leads(X,Y) \rightarrow group(Y)$

$group(X) \rightarrow \exists Y\, works(Y,X)$

$works(X,Y) \rightarrow member(X)$

$chase(D,\Sigma) = D \cup ?$

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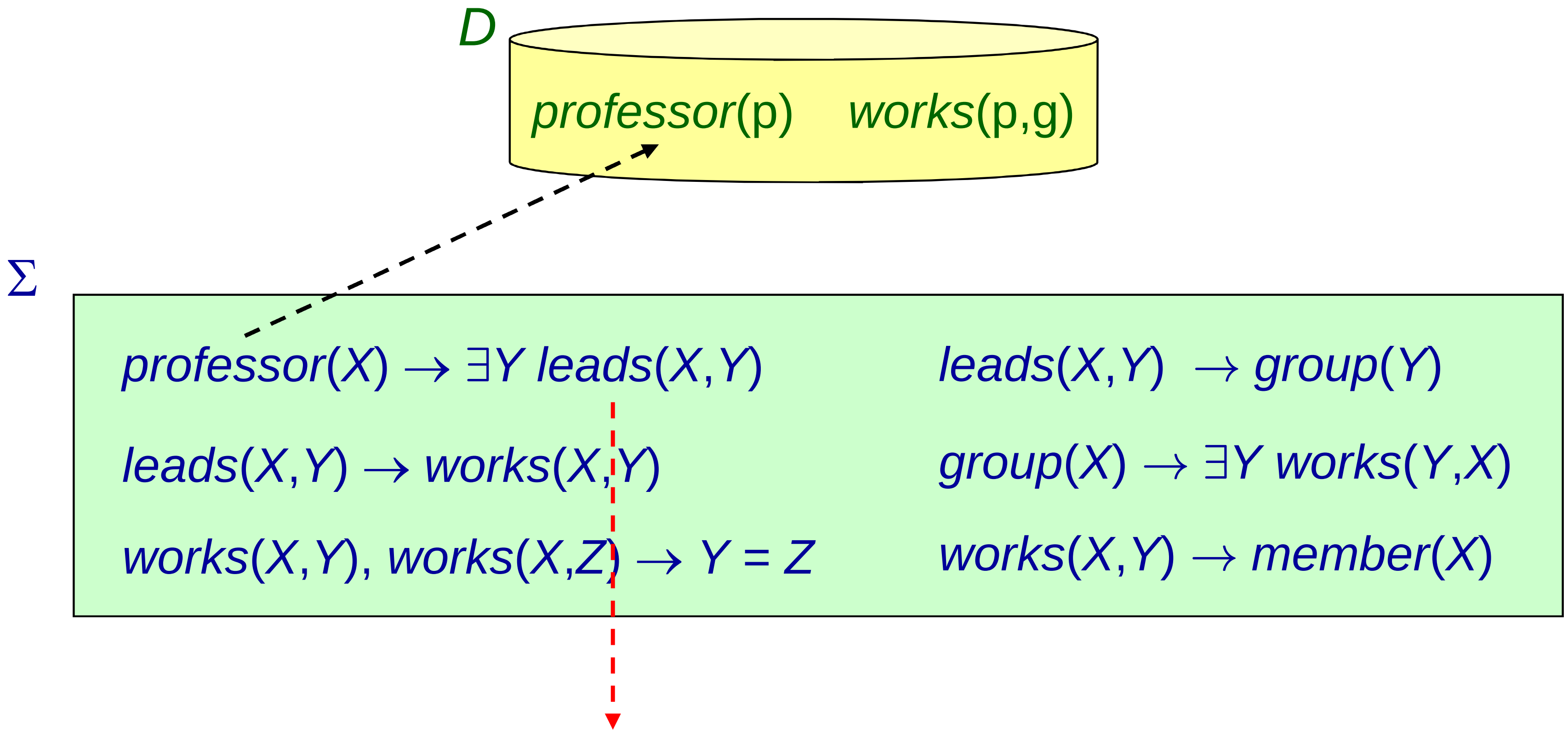

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$chase(D,\Sigma) = D \cup \{leads(p,z_1)$

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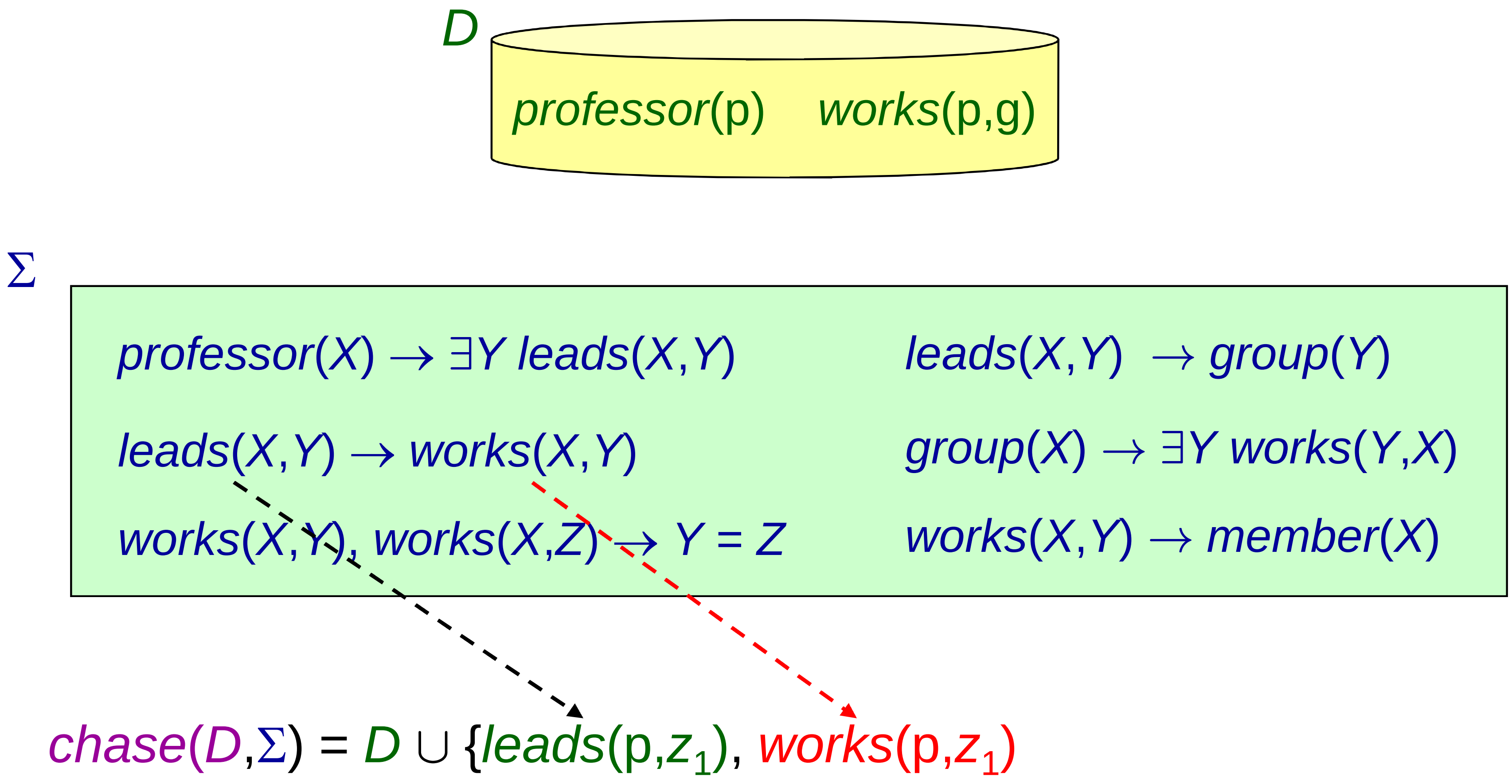

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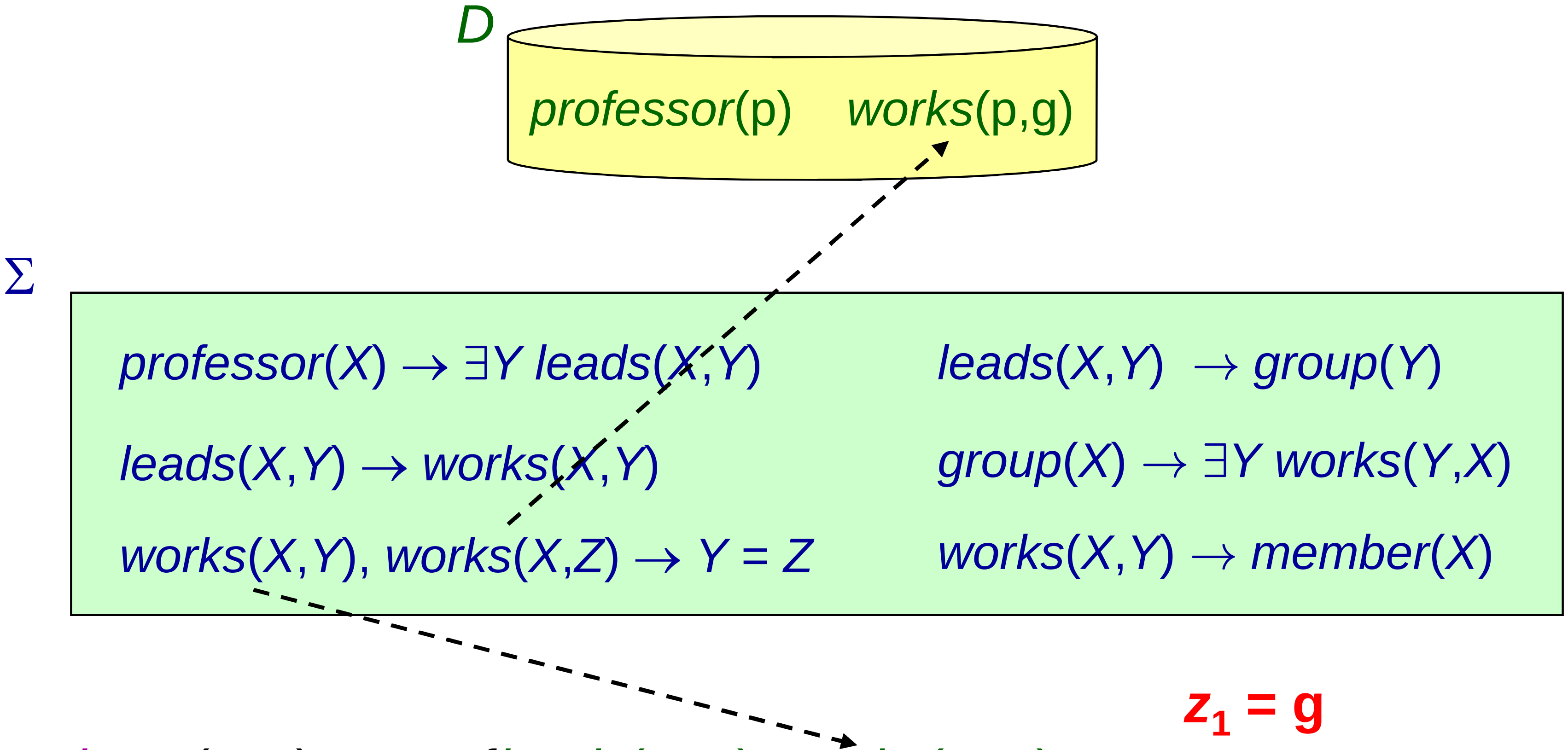

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$\mathbf{z_1 = g}$

$chase(D,\Sigma) = D \cup \{leads(p,z_1),\ works(p,z_1)\}$

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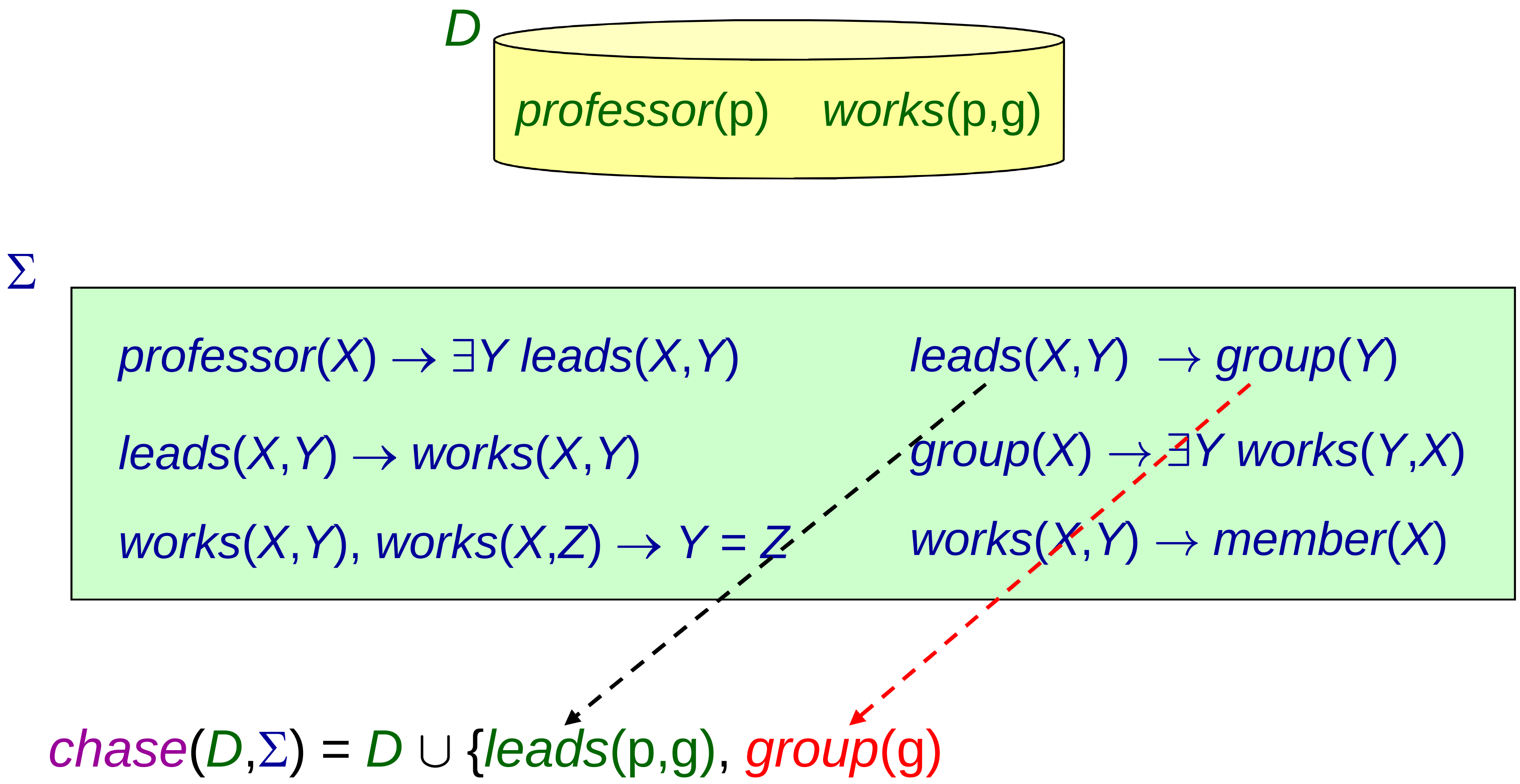

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$chase(D,\Sigma) = D \cup \{leads(\mathrm{p},\mathrm{g}), group(\mathrm{g}), \ldots$

infinite instance

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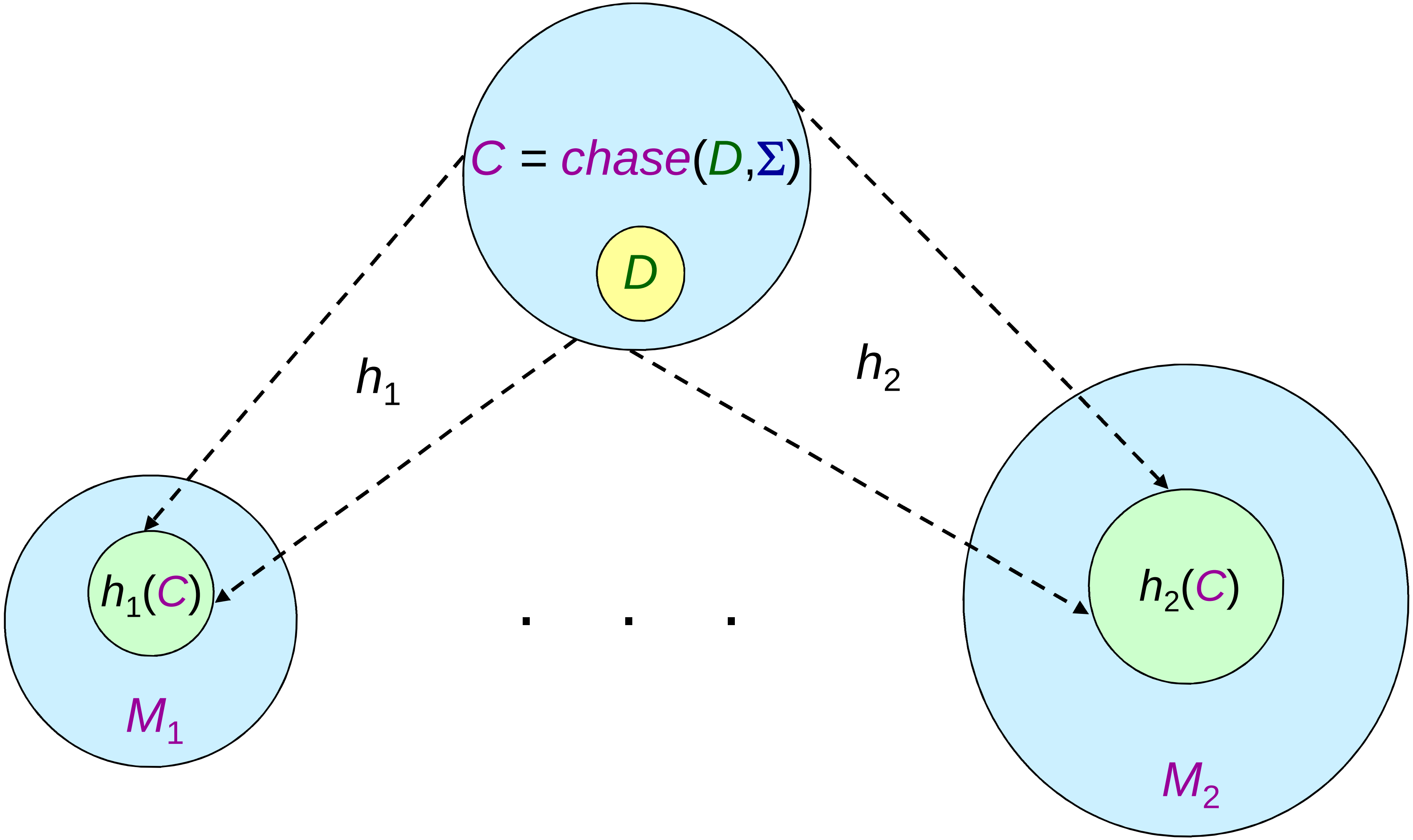
… also may fail due to hard violations of EGDs (equating constants)

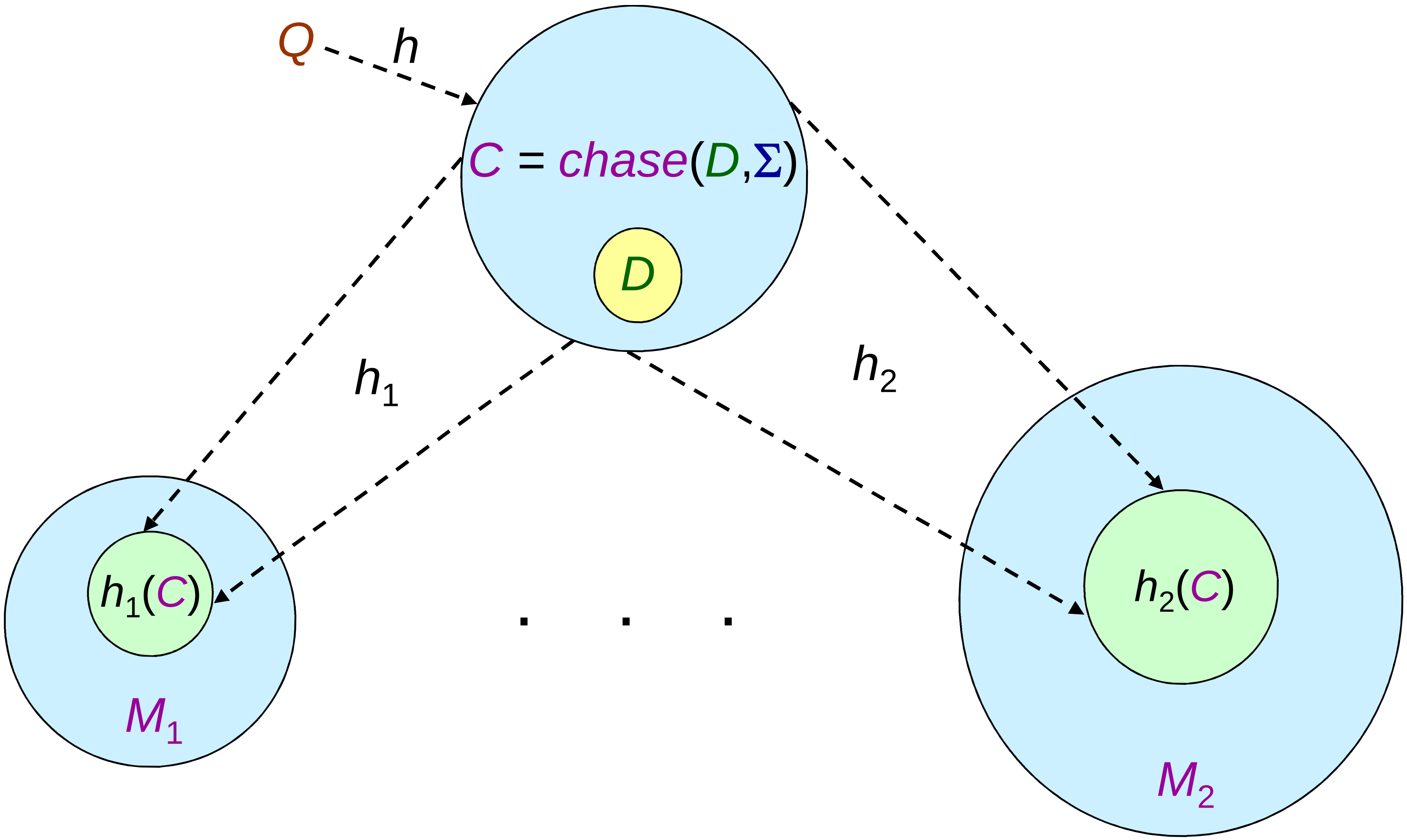
Chase: A Universal Model



$$\forall M\ (M \vDash D \cup \Sigma \quad \Rightarrow \quad chase(D,\Sigma) \dashrightarrow M)$$

[see, e.g., Deutsch, Nash & Remmel, PODS 08]

Query Answering via Chase



$D \cup \Sigma \models Q \quad \Leftrightarrow \quad \mathit{chase}(D,\Sigma) \models Q$

[see, e.g., Deutsch, Nash & Remmel, PODS 08]

Separability

- Consider a set $\Sigma = \Sigma_T \cup \Sigma_E$

TGDs EGDs

- Σ is separable if, for every database D:
  - either $chase(D,\Sigma)$ fails (due to a hard violation)
  - or, for every query Q: $chase(D,\Sigma) \models Q$ iff $chase(D,\Sigma_T) \models Q$

[see, e.g., Calì, Lembo & Rosati, PODS 03]

Separability

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TGDs EGDs

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- or, for every query *Q*: *chase*(D,Σ) $\vDash Q$ iff *chase*(D,Σ_T) $\vDash Q$

NOTE: Chase failure can be checked without adding complexity

[see, e.g., Calì, Lembo & Rosati, PODS 03]

Separability is Desirable

- We can answer queries ignoring EGDs
- Query answering under Linear TGDs only
- AC_0 data complexity of query answering under Linear TGDs

  [Calì, Gottlob & Lukasiewitcz, PODS 09]

Separability is Desirable

– We can answer queries ignoring EGDs

– Query answering under Linear TGDs only

– AC_0 data complexity of query answering under Linear TGDs
[Calì, Gottlob & Lukasiewitcz, PODS 09]

$$\Downarrow$$

We need to define syntactically separable sets of Linear TGDs and EGDs

Non-Conflicting Condition: Intuition

– Given a set $\Sigma = \Sigma_T \cup \Sigma_E$

TGDs EGDs

– For each EGD η of Σ_E

- construct representatives of atoms which are critical after applying η (we use backward resolution)

- check whether the critical atoms are entailed by Σ_T only (we use conjunctive query containment)

Non-Conflicting Condition: Critical Atoms

– Watched variable

  - we apply the EGD $\forall \mathbf{X}\ \Phi(\mathbf{X}) \rightarrow X_1 = X_2$ during the chase
  - the substitution $X_2 \rightarrow X_1$ is applied
  - X_1 is the watched variable

Non-Conflicting Condition: Critical Atoms

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- Affected positions
  - a position is an argument of a predicate, e.g., *employee*[2]
  - at affected positions null values can appear during the chase
  - simple polynomial check

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- Affected positions
  - a position is an argument of a predicate, e.g., *employee*[2]
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  - simple polynomial check

- Critical atom - watched variable at some affected position

Non-Conflicting Condition: Representatives

Σ

$s(X,Y) \rightarrow r(Y,X)$

$p(X) \rightarrow \exists Y\, r(X,Y)$

$p(X) \rightarrow \exists Y\, s(Y,X)$

$t(X,Y) \rightarrow r(X,Y)$

$r(X,Y) \rightarrow s(Y,X)$

$r(X,Y),\, r(X,Z) \rightarrow Y = Z$

watched variable: Y

affected positions: $r[2]$ and $s[1]$

Non-Conflicting Condition: Representatives

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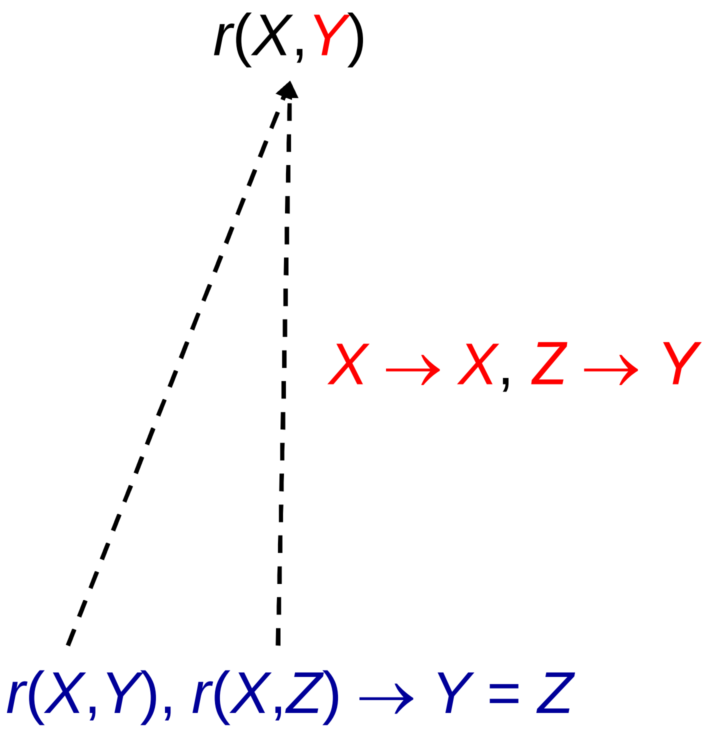

$r(X,Y)$

$X \rightarrow X,\ Z \rightarrow Y$

$r(X,Y),\ r(X,Z) \rightarrow Y = Z$

watched variable: Y

affected positions: $r[2]$ and $s[1]$

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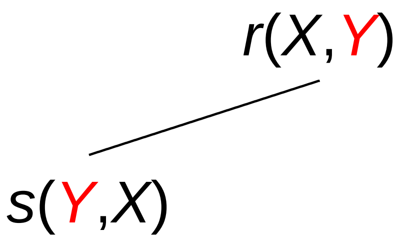

$r(X,Y)$

$s(Y,X)$

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watched variable: Y

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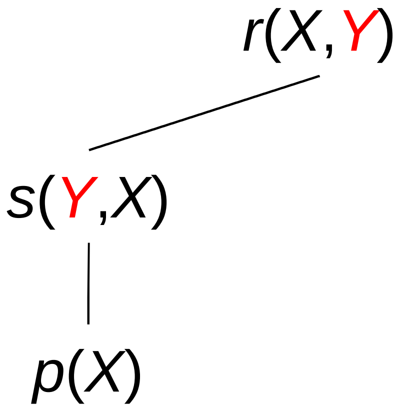

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$p(X) \rightarrow \exists Y\, s(Y,X)$

watched variable: Y

affected positions: $r[2]$ and $s[1]$

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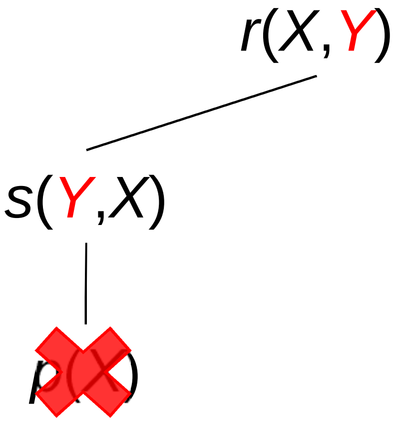

$r(X,Y)$

$s(Y,X)$

$\cancel{p(X)}$

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watched variable: Y

affected positions: $r[2]$ and $s[1]$

Non-Conflicting Condition: Representatives

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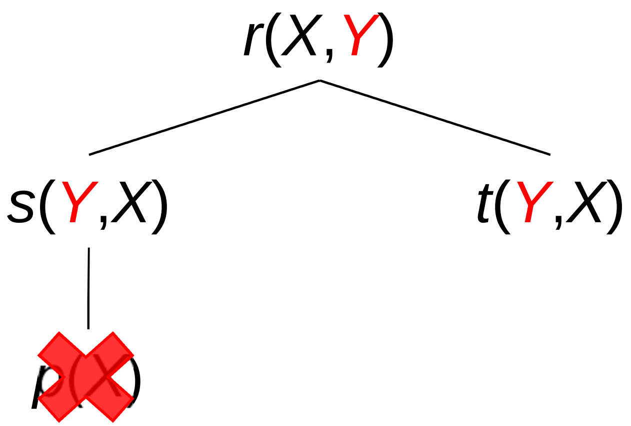

$r(X,Y)$

$s(Y,X)$

$t(Y,X)$

~~$p(X)$~~

$t(X,Y) \rightarrow r(X,Y)$

watched variable: Y

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Non-Conflicting Condition: Representatives

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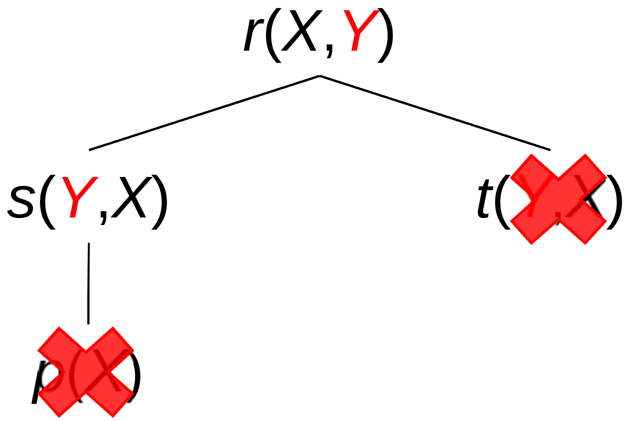

$t(X,Y) \rightarrow r(X,Y)$

watched variable: Y

affected positions: $r[2]$ and $s[1]$

Non-Conflicting Condition: Representatives

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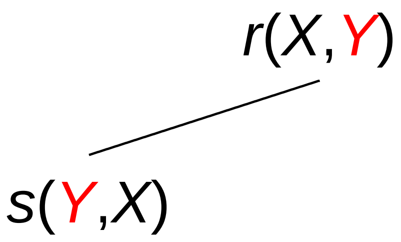


$r(X,Y)$

$s(Y,X)$

– No other representatives can be constructed

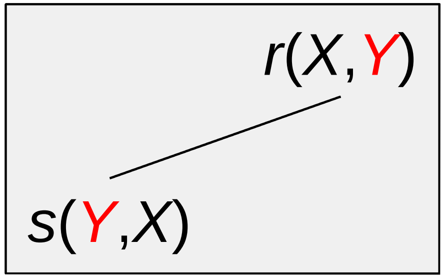
– Finitely many due to linearity

watched variable: Y

affected positions: $r[2]$ and $s[1]$

Non-Conflicting Condition: Containment Check

- The critical atoms are entailed by the TGDs?



- Q_1 is contained in Q_2 under Σ, written as $Q_1 \subseteq_\Sigma Q_2$, if

$$\forall D\ (D \models \Sigma \quad \Rightarrow \quad Q_1(D) \subseteq Q_2(D))$$

Non-Conflicting Condition: Containment Check

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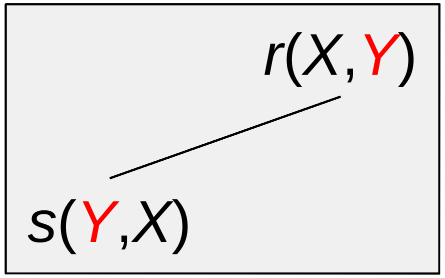

$r(X,Y)$

$s(Y,X)$

- Q_1 is contained in Q_2 under Σ, written as $Q_1 \subseteq_{\Sigma} Q_2$, if

$$\forall D \; (D \vDash \Sigma \quad \Rightarrow \quad Q_1(D) \subseteq Q_2(D))$$

- Construct the conjunctive queries: $Q_1(X,Y) \leftarrow r(X,Y),r(X,Z)$

$$Q_2(X,Y) \leftarrow r(X,Y)$$

$$Q_3(X,Y) \leftarrow s(Y,X)$$

- $Q_1 \subseteq_{\Sigma_T} Q_2 \quad$ and $\quad Q_1 \subseteq_{\Sigma_T} Q_3 \quad \Rightarrow \quad \Sigma$ is non-conflicting

Non-Conflicting Condition vs Separability

Theorem: If Σ is non-conflicting, then it is separable

Proof: $\Sigma = \Sigma_T \cup \Sigma_E$

$$\begin{tikzcd} \forall D: \quad \mathit{chase}(D,\Sigma) \arrow[r, bend left, dashed, "h_1"] & \mathit{chase}(D,\Sigma_T) \arrow[l, bend left, dashed, "h_2"] \end{tikzcd}$$

$$\Downarrow$$

$$\forall Q: \mathit{chase}(D,\Sigma) \vDash Q \quad \text{iff} \quad \mathit{chase}(D,\Sigma_T) \vDash Q$$

Final Considerations

- Identifying non-conflicting sets is PSPACE-complete
  - not harder than query answering under Linear TGDs

    [implicit in Johnson & Klug, JCSS 84]

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– Identifying non-conflicting sets is PSPACE-complete

  - not harder than query answering under Linear TGDs

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– Query answering under non-conflicting sets of Linear TGDs and EGDs is

  - in AC_0 w.r.t. data complexity

    [implicit in Calì, Gottlob & Lukasiewitcz, PODS 09]

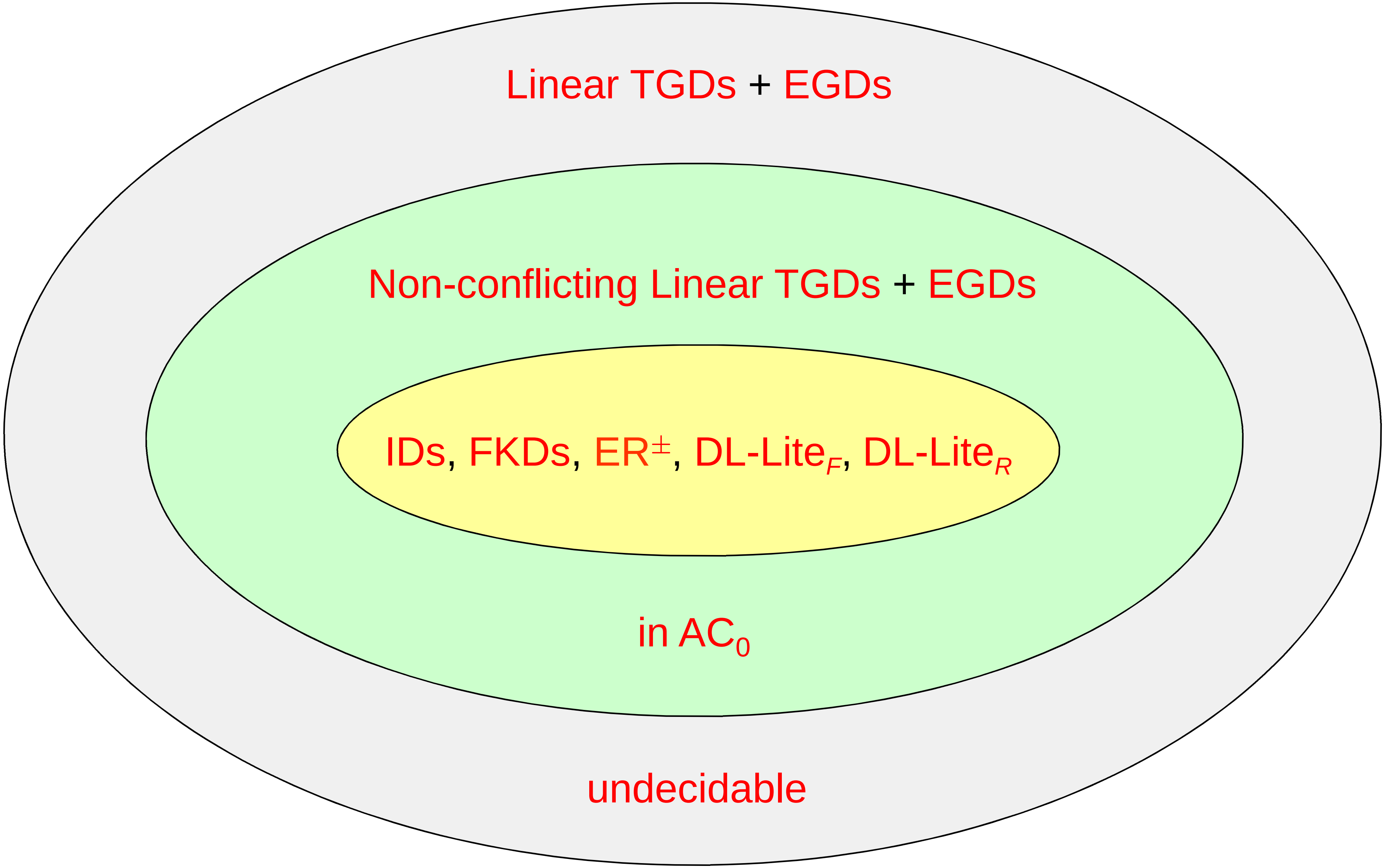
  - PSPACE-complete w.r.t. combined complexity

    [implicit in Johnson & Klug, JCSS 84]

  - same complexity even if we add negative constraints of the form

$$\forall \mathbf{X}\ \Phi(\mathbf{X}) \rightarrow \bot$$

Overview



Linear TGDs + EGDs

Non-conflicting Linear TGDs + EGDs

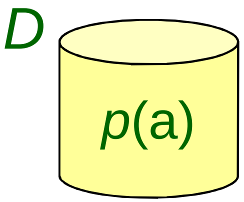
IDs, FKDs, $ER^{\pm}$, DL-$Lite_{F}$, DL-$Lite_{R}$

in AC_0

undecidable

Thank you!

The Key Property



D

$p(\mathrm{a})$

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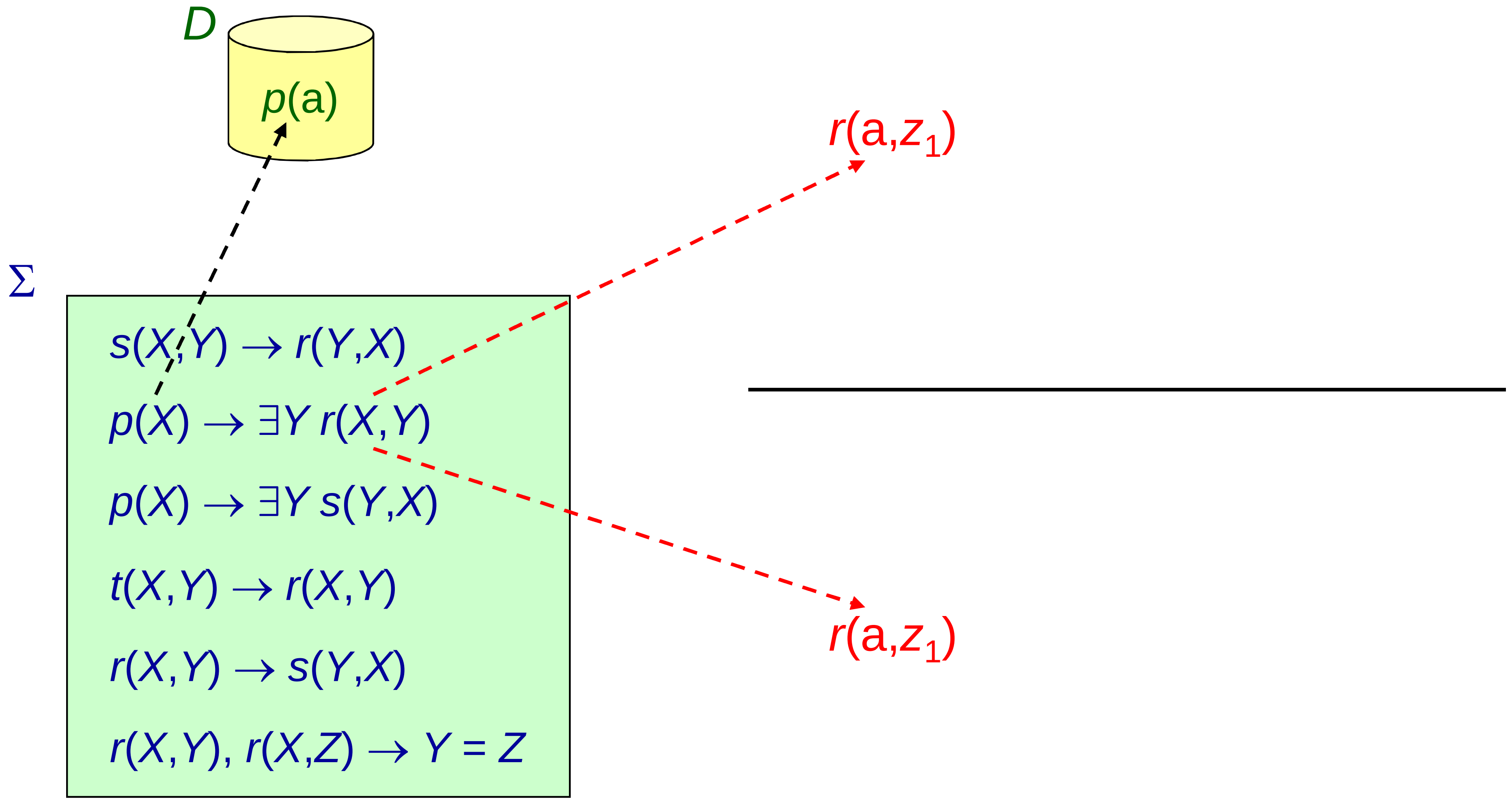
$r(X,Y),\, r(X,Z) \rightarrow Y = Z$

chase under TGDs and EGDs

---

chase under TGDs only

The Key Property



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$r(\mathrm{a},z_1)$

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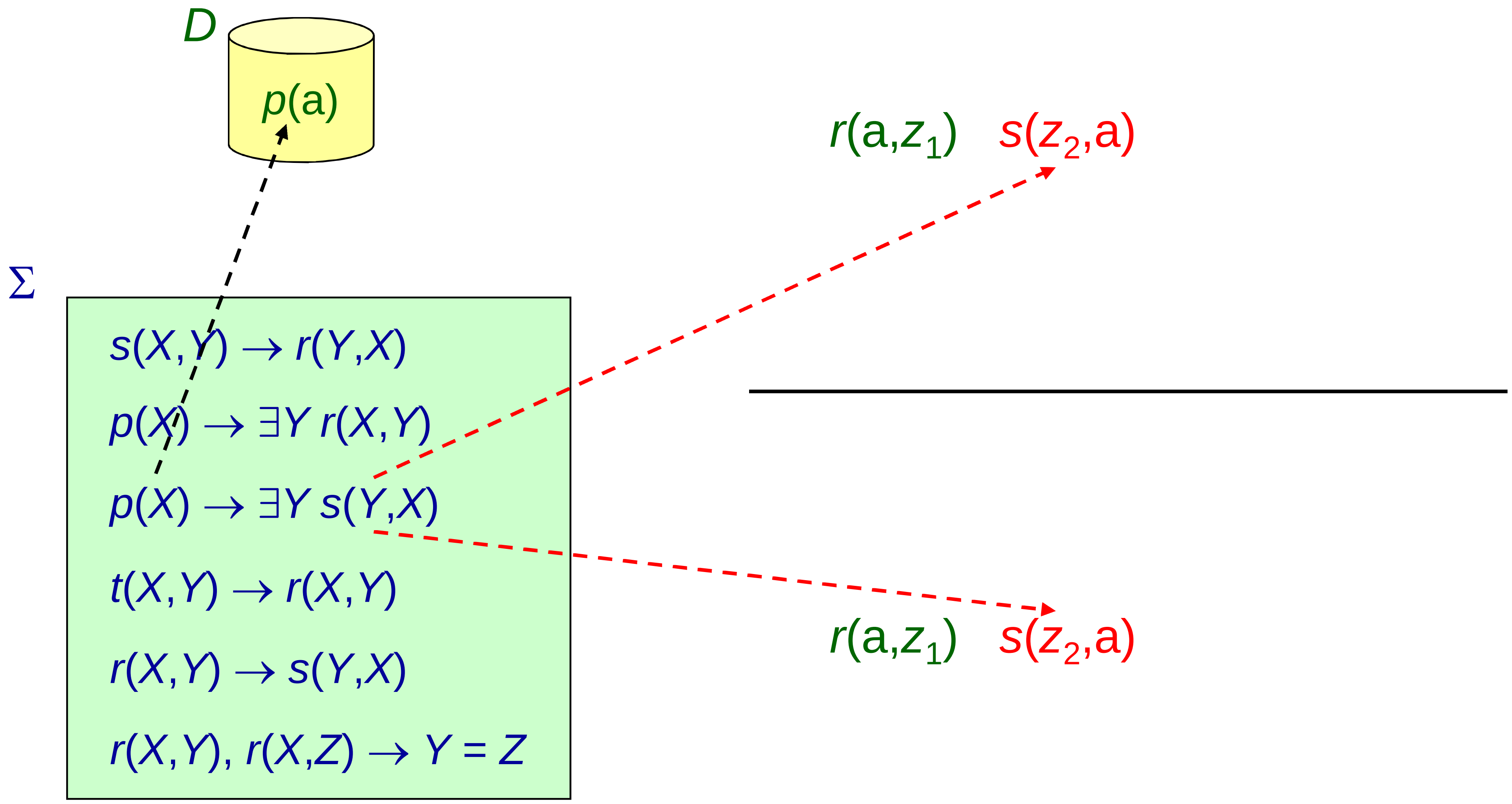
The Key Property



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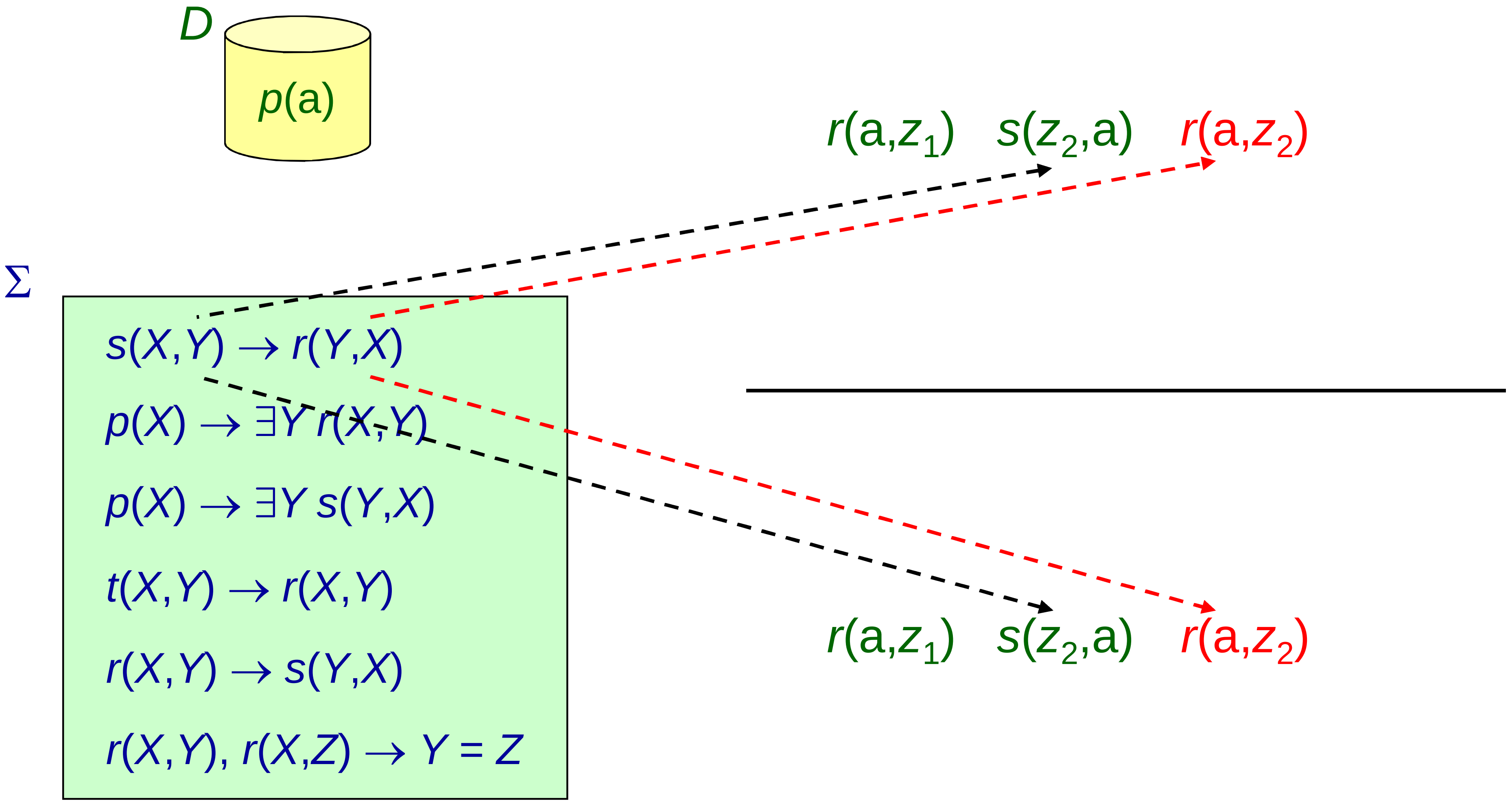
The Key Property



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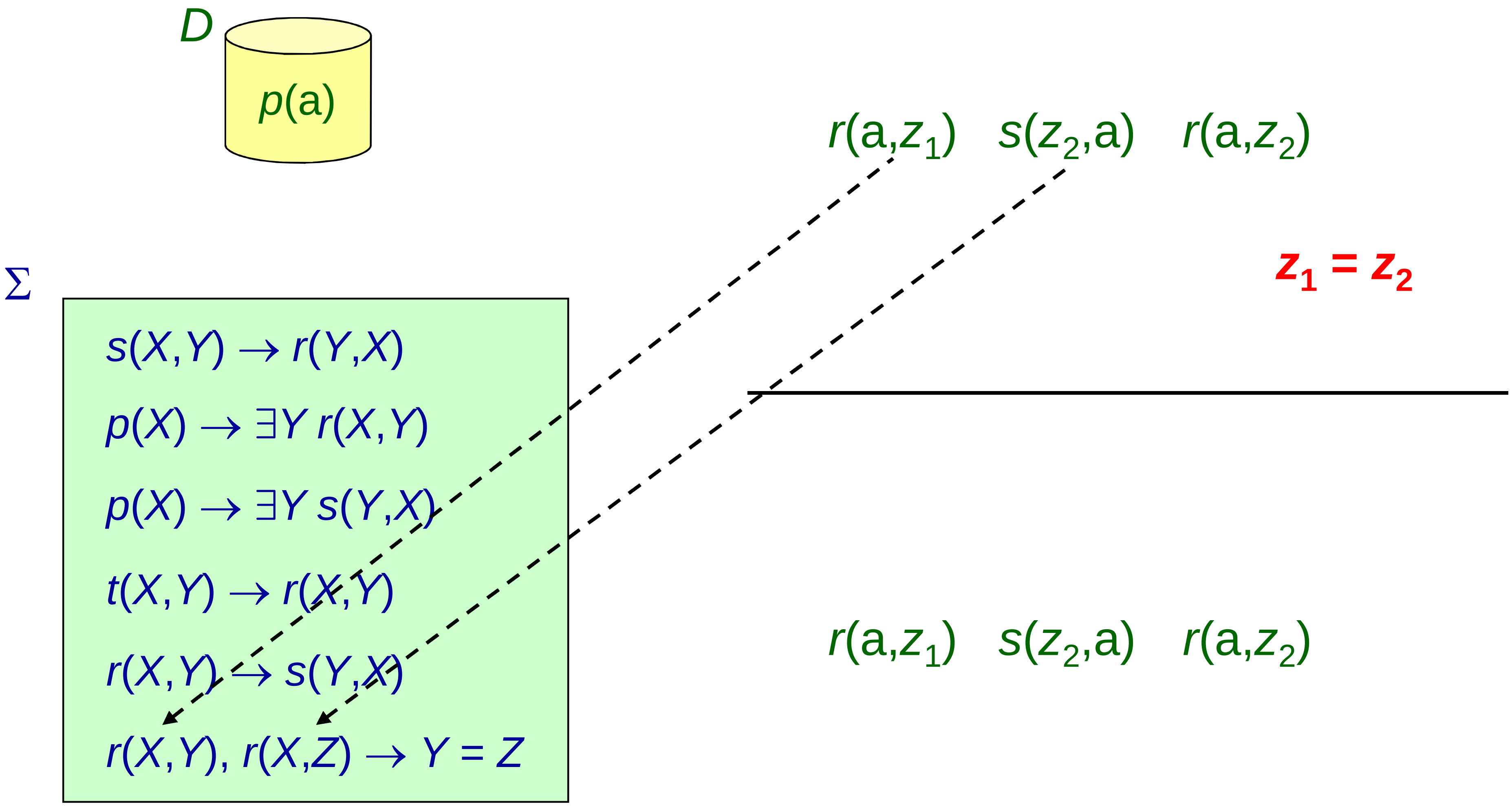

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$\boldsymbol{z_1 = z_2}$

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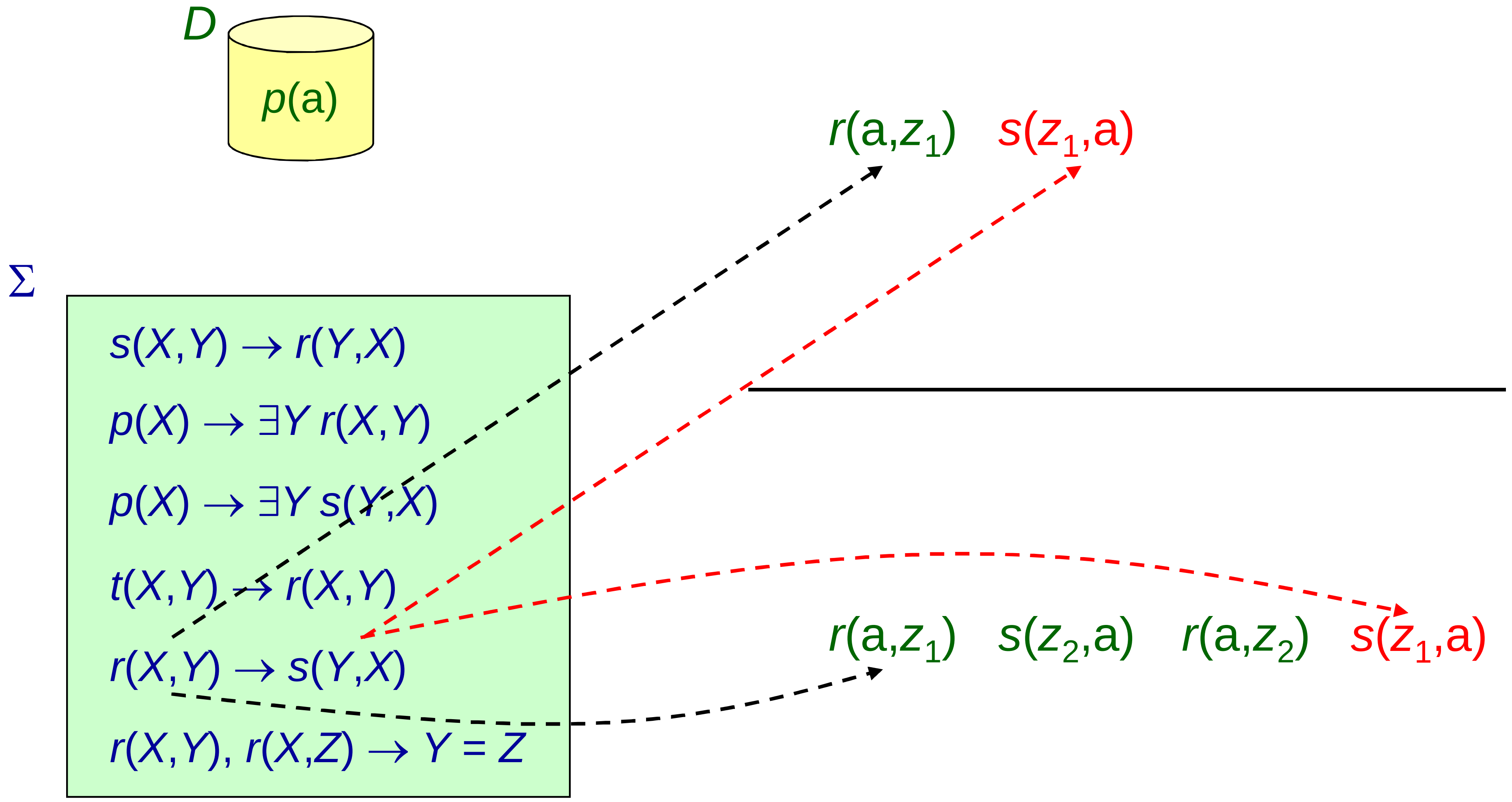

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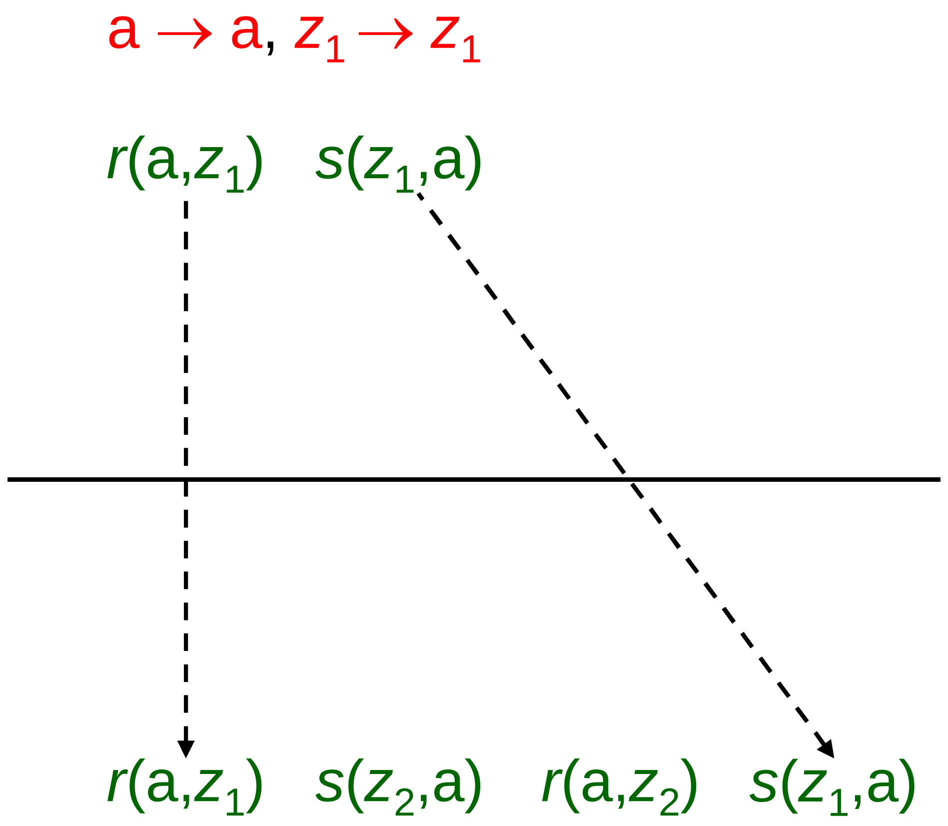

$\mathrm{a} \rightarrow \mathrm{a},\ z_1 \rightarrow z_1$

$r(\mathrm{a},z_1)\quad s(z_1,\mathrm{a})$

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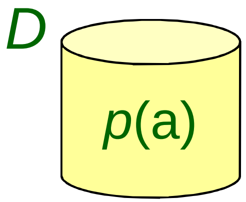

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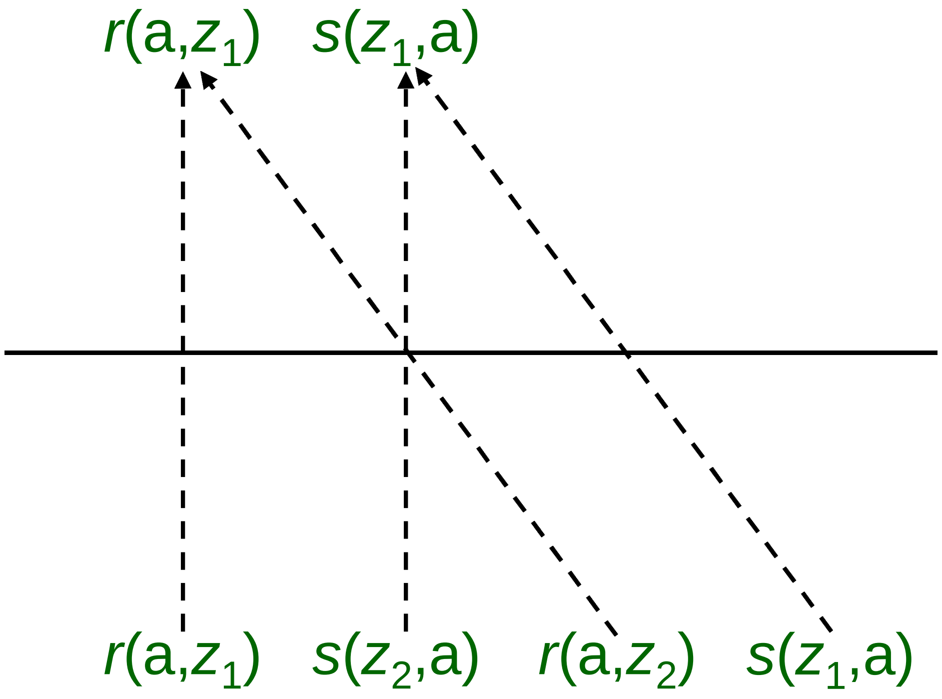

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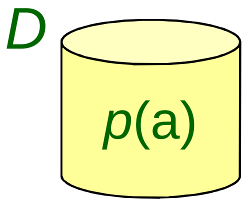
$\mathrm{a} \rightarrow \mathrm{a},\ z_1 \rightarrow z_1,\ z_2 \rightarrow z_1$

The Key Property

D



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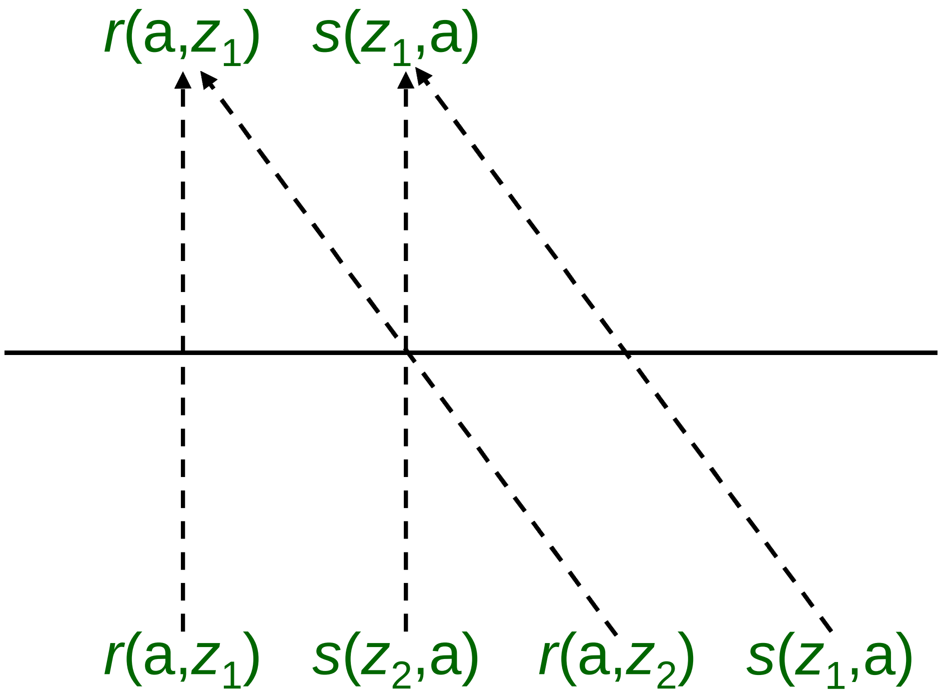

$r(\text{a},z_1)$ $s(z_1,\text{a})$

$r(\text{a},z_1)$ $s(z_2,\text{a})$ $r(\text{a},z_2)$ $s(z_1,\text{a})$

The two chases are

homomorphically equivalent

Non-Conflicting Condition: Representatives

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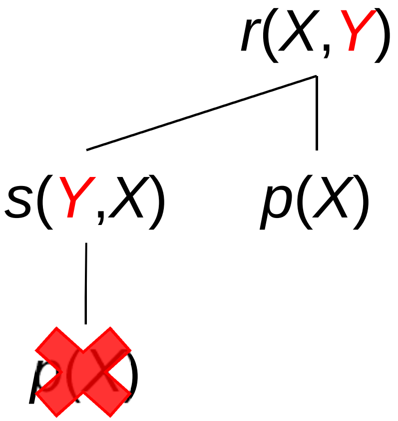

$r(X,Y)$

$s(Y,X)$ $p(X)$

$\xcancel{p(X)}$

$p(X) \rightarrow \exists Y\, r(X,Y)$

watched variable: Y

affected positions: $r[2]$ and $s[1]$

Non-Conflicting Condition: Representatives

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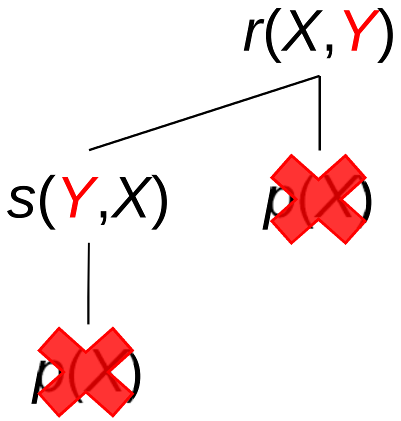

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